

# Rapid Frequency-Domain Modeling Methods for Unmanned Aerial Vehicle Flight Control Applications

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Modeling of the flight dynamics of unmanned aerial vehicles (UAVs) poses unique challenges that are not present with manned aircraft. The use of analytical modeling methods based on first principles is often difficult for UAVs because of short design cycles, reduced development costs, and many unconventional designs. Also, without the need to carry a pilot, UAVs are often much smaller and lighter than manned aircraft. The lower weights and inertias result in higher natural frequencies and quicker vehicle responses requiring high bandwidth dynamics models. Frequency-domain system identification is especially well suited to the modeling of UAVs. With the availability of flight hardware early in many UAV programs, dynamic response models of the vehicle can be identified and validated rapidly with flight data. The system identification method also allows for rapid updating of vehicle response models as physical changes are made to the vehicle configuration. The use of frequency-domain system identification in the development and operation of a number of UAV programs is discussed. The example aircraft programs include Northrop Grumman's Fire Scout vertical takeoff unmanned air vehicle demonstrator based on the Schweizer 300 helicopter; the broad-area unmanned responsive resupply operations UAV based on Kálmán's twin-rotor K-MAX helicopter; AeroVironment's Pathfinder solar-powered stratospheric research aircraft; Yamaha's R-50 small-scale helicopter; and the class of small-scale ducted fan vehicles developed separately by Allied Aerospace (formerly Micro Craft) and Honeywell.

## Nomenclature

$F$	= state matrix
$G$	= control matrix
$H$	= output state matrix
$J$	= output control matrix
$L$	= length, body roll moment
$M$	= body pitch moment
$M$	= descriptor state matrix
$N$	= scale ratio, body yaw moment
$P, Q, R, p, q, r$	= body angular rates
$u, v, w$	= vehicle body velocities
$u$	= control vector
$x$	= state vector
$y$	= output state
$y$	= output vector
$Z$	= body vertical force
$\delta_{col}$	= collective stick
$\delta_{lat}$	= lateral stick
$\delta_{lon}$	= longitudinal stick
$\delta_{ped}$	= pedal control input
$\tau$	= time delay

$\phi, \theta$	= body Euler attitudes
$\omega$	= frequency

## Introduction

UNMANNED aerial vehicles (UAVs) have an ever increasing utility in both civil and military settings for a wide range of applications and missions. UAVs are often tasked to fly in difficult weather conditions, in the presence of obstacles, and close to populated areas much like manned aircraft, but they have to do this autonomously. Like manned aircraft, a key requirement for the development and validation of guidance and control algorithms for UAVs is an accurate model of the vehicle dynamic response to control inputs and disturbances. However, the modeling of the flight dynamics of UAVs poses some additional challenges that are not present with manned vehicles.

For UAVs, it is often difficult to develop and validate analytical flight dynamics models based on first principles where the aerodynamics, structures, and controls are modeled explicitly. This is because the short design cycles and reduced cost of many UAV programs do not allow for the development of these analytical models. Also, many unconventional UAV designs<sup>1</sup> have been proposed and flown, and these new designs are not easily modeled using analytical methods. System identification methods provide an alternative to analytical methods and are well suited to the modeling of UAVs. With many UAV programs, flight hardware is available early in the program, and with the flight vehicle available, dynamic response models of the vehicle can be extracted and validated rapidly from flight data. The system identification methods also allow for rapid updating of the vehicle response models as changes are made to the vehicle by repeating flight tests.

The focus of this paper is on the use of frequency-domain system identification for UAV applications. Frequency-domain system identification methods have a number of advantages over time-domain identification methods that make them particularly relevant to UAVs. Some of the advantages of frequency-domain methods are they can be used with vehicles that exhibit unstable as well as stable dynamics, they do not require a model structure for the identification of frequency responses (nonparametric models), they provide direct information on flight control metrics including stability margins and control crossover frequencies, and they produce a coherence function that is a direct measure of linearity and data quality.

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This paper summarizes the experience gathered at the U.S. Army/NASA Rotorcraft Division at NASA Ames Research Center in the use of frequency-domain system identification for the modeling of UAV flight dynamics. The particular frequency-domain system identification method used is implemented in the CIPHER<sup>®</sup> tool.<sup>2</sup> In the first sections, the unique challenges of UAV modeling are examined and the example vehicles are described. In the next sections, an overview is provided of the rapid frequency-domain system identification procedure as implemented in CIPHER, and how the dynamics models are used in control system development is described. This is followed by a discussion of the flight-test requirements and procedures for system identification. In the final sections, the specifics are described of the dynamics models (both vehicle dynamics models and sub-system dynamics models) generated during the system identification, from nonparametric frequency responses to transfer function models and finally to higher-order state-space representations of the vehicle dynamics.

### Challenges of UAV Modeling

Without the need to carry a pilot, many UAVs are much smaller and lighter than manned vehicles. The lower weight and inertias of these vehicles result in higher natural frequencies of the vehicle dynamics and quicker vehicle response times. The dynamic characteristics of small-scale vehicles are related to the dynamics of larger (manned scale) vehicles by Froude scaling. Froude scaling ensures common ratios between the inertia-to-gravity forces and the aero-to-gravity forces for vehicles of different sizes. The geometric and dynamic characteristics of different size vehicles are related by a set of similarity laws<sup>3</sup> and are based on a scale ratio,  $N$  (where a scale ratio of  $N = 4$  refers to one-fourth scale). Thus, length

$$L_{\text{small-scale}} = L_{\text{full-scale}}/N \quad (1a)$$

and frequency

$$\omega_{\text{small-scale}} = \omega_{\text{full-scale}}\sqrt{N} \quad (1b)$$

When Froude scaling is used, the vehicle natural frequency  $\omega$  increases by the square root of the scale ratio. Thus for a one-fourth scale vehicle, the natural frequency increases by a factor of two. This in turn dictates the need for higher bandwidth flight control systems, which require higher bandwidth dynamics models for analysis. Additionally, small-scale UAVs typically have lower available weight and volume for the flight control computer and sensors. This results in compromised systems, relative to manned-scale vehicles: flight control computers that have limited power and speed, sensors that are noisy and have lower performance, and actuators that are slower and less precise. To obtain the greatest potential benefit of flight control synthesis in the presence of these subsystem limitations again requires highly accurate models of the vehicle flight dynamics.

Many existing UAVs exploit only a modest portion of their available flight envelope. This limitation is mainly due to control system designs that are highly conservative. For higher bandwidth control systems to exploit a larger portion of the available flight envelope, dynamic models that are valid over a wide frequency range and in a range of different operating conditions are required. Such models can be obtained using frequency-domain system identification techniques.

Rotorcraft and other vertical takeoff and landing designs are seeing increased use as UAVs due to their ability to operate from confined and unimproved areas and their high degree of maneuverability in hover and slow-speed flight. The inherent dynamics of rotorcraft makes the development of rotorcraft UAVs (RUAVs) especially difficult. Unlike most fixed-wing UAVs, the bare airframe of an RUAV is unstable, has a high degree of interaxis coupling, has large response delays associated with the rotor, and has a large response variation with flight condition. Additionally, rotorcraft dynamics include lightly damped fuselage and rotor modes. Considerable knowledge of rotorcraft flight dynamics is required to build dynamics models from first principles to attain sufficient accuracy for flight control synthesis.

### Description of Example Vehicles

This paper includes details of the frequency-domain system identification method as it has been successfully applied to a number of different UAV programs. The examples presented include both fixed- and rotary-wing UAVs, some of which are adapted from manned vehicle designs and some of which are entirely new unmanned designs. Figure 1 shows the UAVs described in this paper along with the key challenges of each from the perspective of system identification and flight control law development.

An example of a fixed-wing UAV is the Pathfinder platform that is a solar-powered flying wing that has been able to attain an altitude of over 80,000 ft (Ref. 4). This vehicle was developed by AeroVironment for NASA's Environmental Research Aircraft and Sensor Technology program. From a flight dynamics modeling perspective, the particular challenges with this vehicle are the very flexible and

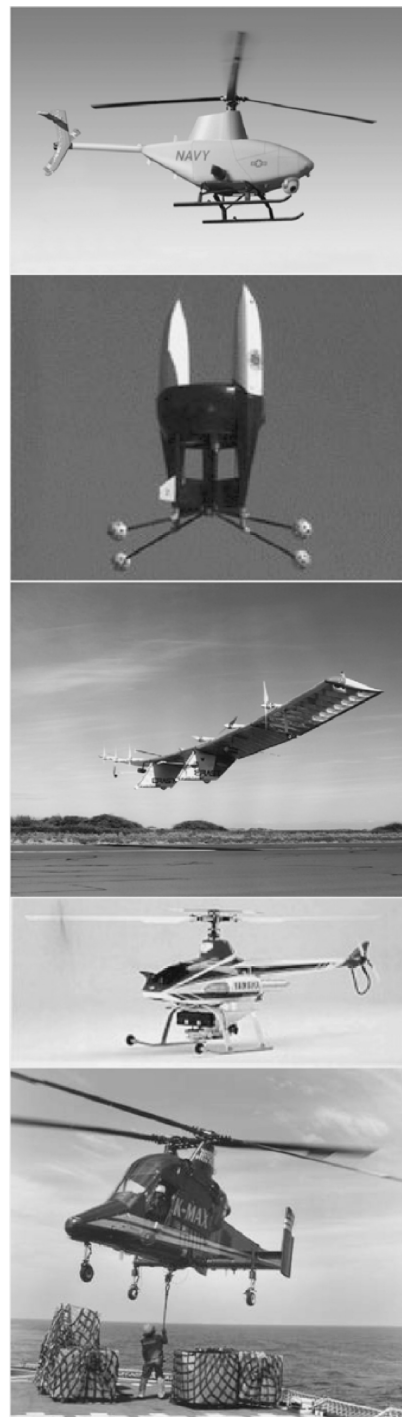


Fig. 1 Example UAV programs.

light wing structure, the slow flight speed (cruise at  $\sim 21$  mph indicated airspeed) where Reynolds number effects are important, and the varying atmospheric environment as the vehicle climbs to and descends from high altitudes. This makes the vehicle flight dynamics very difficult to model accurately analytically because of the dominance of structural and aerodynamic modes on the low-frequency response of the vehicle. Frequency-domain system identification was used to assist in the envelope expansion to progressively higher altitudes, to study the effects of altitude on the vehicle response, to characterize the actuator dynamics, and as a means of validating and refining the analytical vehicle simulation model.

Two RUAV programs that are based on vehicles that were originally designed for piloted operations and have been adapted to unmanned configurations are also examined. The first is the BURRO program that used a Kaman K-MAX helicopter that was modified for remotely piloted and autonomous flight.<sup>5,6</sup> The K-MAX features overlapping, intermeshing twin main rotors with no tail rotor in an arrangement known as a "synchropter." The unique challenge with this vehicle was associated with the ability to lift a payload weight as much as the vehicle itself as an externally slung load that produces a difficult two-body dynamics problem. System identification was performed with and without external loads, and the associated dynamic models included states for the external load position as well as those for the flight vehicle. The second example of an RUAV adapted from a piloted aircraft is the Northrop Grumman Vertical Take-off UAV (VTUAV) demonstrator named Fire Scout.<sup>7</sup> Fire Scout is adapted from a Schweizer 333 helicopter that has a three-bladed fully articulated hub (with a tail rotor for antitorque) with a takeoff gross weight of 2550 lb. This vehicle was developed on a very tight development schedule with only four months to design and develop the flight control laws for a fully autonomous flight demonstration. Also no piloted or operator-in-the-loop reversion mode was included during the autonomous demonstration, increasing the reliance on the vehicle modeling. An example of a small-scale RUAV is the Yamaha R-50 that has a rotor diameter of 10 ft and a maximum gross weight of 150 lb (Refs. 8 and 9). This commercial small-scale helicopter was originally developed for remotely piloted crop spraying applications. The helicopter has a two-bladed teetering rotor configuration (with a tail rotor for antitorque) and a stabilizer bar. The key challenge with this vehicle is that the dynamic response is dominated by the rotor system and dynamic models that are of sufficient accuracy for flight control work must explicitly include the dynamics of the coupled rotor-fuselage system and the stabilizer bar.

The final example UAVs described in this paper are the small ducted fan-type vehicles developed separately by Allied Aerospace (then Micro Craft)<sup>10</sup> and Honeywell. These vehicles are characterized by a cylindrical duct surrounding a propeller or fan that rotates around the vehicle's vertical axis. Control is achieved by varying the speed of the propeller and by deflecting control vanes that are mounted downstream of the propeller at the lower end of the duct. There are a number of challenges with this type of UAV that make the task of flight control design and optimization difficult. Most of the mass is concentrated along the vehicle centerline near the center-of-gravity of the body, which results in lower inertias and then high natural frequencies of the vehicle response. The small-scale limits the size and weight of the sensors, actuators and flight control computer and makes them prone to large amounts of drift and noise. The bare airframe of the vehicle is inherently unstable and requires the control system to provide basic stability. Frequency-domain system identification techniques were used in both the Micro Craft and Honeywell programs with the vehicle in the hover condition. Bench test identification of the actuator dynamics was also performed for both vehicles.

For the UAV programs described in this paper, the challenges are summarized as follows. The key challenges with Fire Scout were the tight development schedule to a fully autonomous flight demonstration, on-axis transfer functions identified in 2 weeks, and full state-space models identified within 2 months. The key challenges with the ducted-fan program were the small scale with low mass and inertia properties, fast response due to low inertias, degraded sensor

performance due to size and weight limitations, and unstable bare-airframe dynamics. For Pathfinder, the key challenges were the very light and flexible structure difficult to model using traditional methods, slow flight regime producing complex aerodynamics, and varying atmospheric conditions with altitude. For Yamaha R-50, the key challenges were low body mass and inertias, dynamic response dominated by rotor and stabilizer bar, and that the model must explicitly include equations for rotor and stabilizer bar dynamics. The BURRO key challenges included system identification with and without slung load, models must include rotor and load DOF, and significant sensor and system time delays that were challenging for control performance.

## Control System Development Tools

To facilitate the development of UAV dynamic models and control systems, the U.S. Army/NASA Rotorcraft Division has adapted a set of integrated modeling and control system design and analysis tools under the Control and Simulation Technologies for Autonomous Rotorcraft (COSTAR) initiative, begun in 1998. These tools were originally developed for manned vehicle applications and have been adapted for UAVs and include comprehensive identification from frequency response (CIFER), control designers unified interface (CONDUIT<sup>®</sup>), and real-time interactive prototype technology integration/development environment (RIPTIDE).

CIFER is a frequency-domain system identification tool that generates the dynamic response models that are used in the control law development and real-time simulation.<sup>2</sup> CIFER provides a comprehensive set of system identification tools to obtain these models and has been successfully used in the development of many manned and unmanned vehicles. All of the frequency-domain system identification cases described in this paper were generated using CIFER. CONDUIT<sup>11</sup> provides a single interactive environment for the visualization and optimization of control law performance against available sets of stability, handling qualities, and performance specifications. Finally, RIPTIDE<sup>12</sup> provides a desktop-based simulation environment that makes real-time, visual, full-flight-envelope operator-in-the-loop or autonomous simulation readily available. Furthermore, RIPTIDE provides the ability to interactively change control system parameters in real time and to observe the modified response in a graphical simulation environment, which can lead to more robust designs in a shorter time. The effectiveness of the CONDUIT and RIPTIDE work in the development of UAV flight control systems and autonomous algorithms is directly related to the accuracy and fidelity of the dynamic models of the air vehicle and subsystems. Such models may be created from first principles, or extracted from bench- and flight-test data using system identification.

## Frequency-Domain System Identification

The basic control architecture that is common for manned and unmanned vehicles is shown in Fig. 2. There are dynamics and time delays associated with each of the subsystem components in the block diagram in addition to the dynamics of the vehicle. To perform for control system design and optimization, accurate models of the bare-airframe dynamics and subsystem dynamics are required. System identification is a method for accurate characterization of

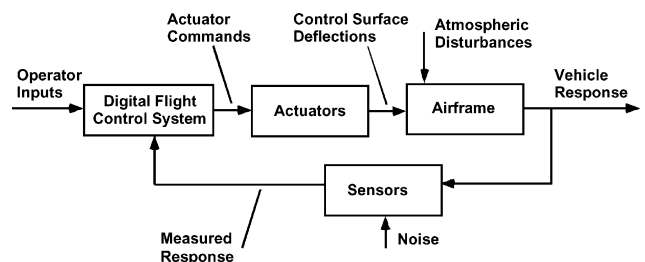


Fig. 2 Common control architecture for manned and unmanned vehicles.

the dynamic response behavior of a complete aircraft, subsystem, or individual component (Fig. 2).

In frequency-domain system identification, the extraction and analysis of dynamic models first involves generating a set of nonparametric frequency responses from a set of time history flight data and then fitting transfer functions and higher-order state-space models to the frequency responses. The goal of this system identification process is to achieve the best possible fit between the flight data and a model that is consistent with the physical knowledge of the vehicle dynamics.

The steps involved in taking a set of flight data through the system identification procedure to a final verified state-space model are 1) frequency-response calculation from time history data, 2) multi-input frequency-response conditioning, 3) multiwindow averaging of the frequency responses, 4) state-space model identification from a set of frequency responses, and 5) time-domain verification of the state-space model. (Details may be found in Ref. 2.)

### Flight-Test Procedure

The collection of a set of high-quality test data is an essential element of a successful identification of any dynamic system because the resulting dynamic models are only as good as the original flight data. There are specific requirements on the system identification flight tests and vehicle configuration that should be followed to ensure a good clean set of test data. These requirements include those on the flight-test technique, the instrumentation (sensors and controls) including filtering and sample rates, the particular maneuvers flown (system identification and verification maneuvers), and the environmental conditions.

#### Sensor Instrumentation

The sensor requirements for system identification of UAVs are somewhat dependent on the application of the identified model. The minimum sensor requirements would include angular rate gyros in each axis. Greater accuracy in the identified models would be achieved with the addition of accelerometer measurements in each axis, vertical gyro measurements for independent attitude information, magnetometer measurements for vehicle heading, air boom data for airspeed, and GPS data for vehicle location. The filtering of all data channels should be documented at the time of flight tests. With reference to Fig. 2, the measured vehicle response differs from the bare-airframe vehicle response because of the dynamics and filtering of the sensors. When the dynamics of the sensors is isolated, the actual bare-airframe vehicle response can be inferred from the measured responses.

#### Control Instrumentation

For vehicle control instrumentation, it would be ideal for the control surface and actuator deflections to be recorded, in addition to the operator commands. This is not always possible, particularly with small-scale UAVs where space and measurement limitations make it impractical or impossible to instrument the control surfaces. For these cases, the actuator commands are sufficient as long as the actuator dynamics and the control linkages are known. The actuator dynamics can be isolated in ground tests with an input of the actuator command and an output of the actuator response or the control surface deflection (shown in Fig. 2). The actuator dynamics can then be accounted for in the identification of the bare-airframe dynamics by isolating the dynamics of the actuators.

#### System Identification Maneuvers

There are two types of maneuvers that are flown as part of a system identification flight-test program. These are system identification and verification maneuvers.

System identification maneuvers are used to gather test data that capture all of the important dynamics of the vehicle in a specified frequency range with the important aspect being the frequency content of the measured data. Ideally, a frequency sweep technique would be used, but multistep maneuvers (described in the next section) may also provide the amount of frequency content that is required

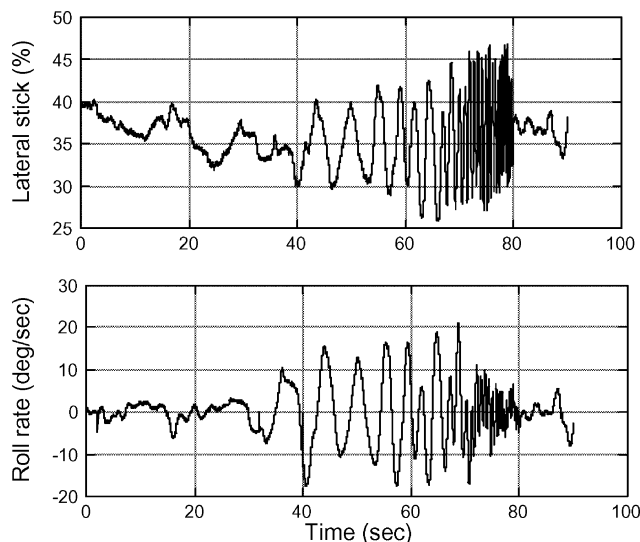


Fig. 3 Example frequency sweep time history for Fire Scout in hover.<sup>7</sup>

for system identification. Details of frequency sweep flight testing are provided in Ref. 13. Figure 3 shows an example of a piloted frequency sweep flown on Fire Scout in hover. The frequency range of the excitation must be tailored to the dynamics of the particular vehicle and the required bandwidth of the final model. For instance, for the BURRO UAV,<sup>5,6</sup> sweep maneuvers covered the range from 0.05 to 2 Hz on each control axis to include all of the important dynamics for the development of moderate bandwidth flight control laws.

For small-scale UAVs, the vehicle bandwidths are higher as a result of the lower vehicle mass and inertias. To account for these higher frequencies, the excitation frequencies required for the flight tests must be adjusted accordingly. This often proves to be difficult with a vehicle that responds quickly, and it is often difficult to perform frequency sweep maneuvers. Instead, multi-step maneuvers can be flown and system identification performed with the resultant flight data. Results with a small ducted-fan type vehicle have shown that multistep data have sufficient frequency content to allow accurate dynamics models to be identified.

#### Model Verification Maneuvers

The second type of maneuvers that are flown as part of a system identification flight-test program are verification maneuvers that typically consist of doublets or multistep inputs. A doublet consists of a positive step input in a single control from trim, which is followed by a negative step of the same magnitude. Finally the control is returned to the nominal location. Multistep maneuvers consist of additional positive and negative control step inputs of varying magnitudes and times. Figure 4 shows a 3211 multistep maneuver flown on the R-50 in hover. Model verification maneuvers are flown in each axis to obtain data that are used for the time-domain verification of the identified models.

#### UAV Flight-Test Techniques

There are a number of different techniques for performing the flight maneuvers required for frequency-domain system identification. The tests can be performed piloted (both with the pilot aboard and on the ground) or with the maneuvers generated automatically. For the BURRO and Fire Scout vehicles, which are unmanned vehicle platforms adapted from original manned vehicles, the system identification flight-tests were flown with a pilot aboard. When flight testing with a pilot aboard, it is important to ensure that the flight-test vehicle properties (weight, inertias and center-of-gravity location) are the same as those of the UAV. If the vehicle properties are the same, then the dynamic response of the bare airframe of the vehicle (Fig. 2) will be the same regardless of whether the maneuvers are generated by an onboard pilot, a remote pilot, or automatically.

For the R-50,<sup>8,9</sup> all of the vehicle maneuvers were flown remotely with the pilot on the ground. In this case, it was difficult for the pilot on the ground to perform the maneuvers accurately because the typical visual and motion cueing with the pilot aboard is not available. Also the vehicle location with respect to the remote pilot is constantly changing, especially in forward flight, which adds to the difficulties. One way to mitigate these problems with UAVs is to have the pilot practice the system identification tests, first in simulation and then in flight, to ensure that the maneuvers are flown correctly.

The Pathfinder program<sup>4</sup> used a different technique where frequency sweep maneuvers were performed automatically onboard the vehicle by an operator command. Figure 5 shows a typical time history for an automated frequency sweep flown in Pathfinder.

### Air Vehicle Model Identification

The goal of the dynamic model identification is to generate models that accurately characterize the dynamic response behavior of the bare airframe of the vehicle. There are a number of different forms that these dynamic models can take with varying levels of complex-

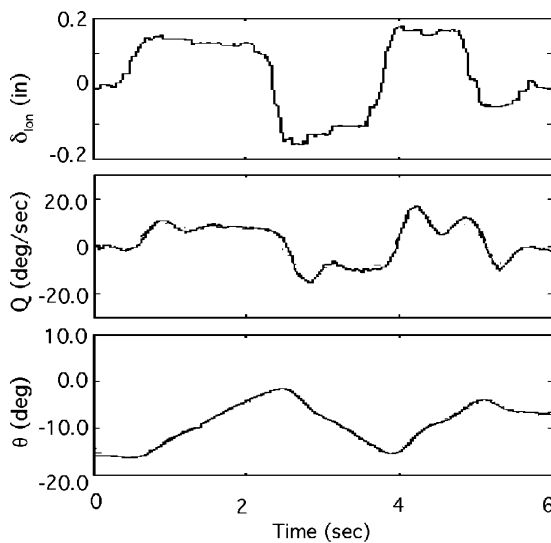


Fig. 4 Example doublet time history for the R-50 in hover.<sup>9</sup>

ity and investment in time and cost. These models are 1) nonparametric frequency response models and 2) parametric models, which include a) transfer-function models and b) state-space models.

The advantages and disadvantages of each of these types of identified models are described and illustrated with some UAV examples in the following paragraphs.

### Nonparametric Frequency Response Models

The simplest dynamic models are nonparametric frequency responses that describe the magnitude and phase response characteristics of an output channel as a function of the frequency of an input. These nonparametric models can be generated in near real time and require little additional investment beyond the collection of the flight-test data.

Identification of single-input/single-output nonparametric frequency responses was performed on the Pathfinder<sup>4</sup> vehicle program in near real time<sup>14</sup> to support envelope expansion as the aircraft climbed to higher altitudes. Automated frequency sweeps were flown at each 10,000-ft interval and on-axis frequency responses were identified. These frequency responses were used to extract metrics such as the crossover frequency and the gain and phase margins to determine the feedback stability and noise amplification properties of the vehicle control system. A limited set of control law gains could be adjusted with the vehicle in flight to provide adequate levels of stability margin. This was an important part of the envelope expansion of the Pathfinder vehicle because the stability margins could be evaluated in real time with the vehicle in flight, the control gains adjusted in real time and decisions made whether to continue with the ascent of the vehicle.

The same automated sweep and frequency-response identification technique can be used for very long endurance flights to detect slow system degradations (such as sensor and actuator drift) that could not otherwise be detected. This type of automated sweep technique also has application in health and usage monitoring systems, where the degradation in system performance (slow or fast) could be evaluated by performing preprogrammed frequency sweep maneuvers at periodic intervals.

The near real-time identification of frequency responses can also be used during flight tests to identify problems with the measured vehicle response due to problems with instrumentation, sensors, or controls.<sup>14</sup> These issues can be quickly identified and resolved with less impact on the test schedule than would have previously been the case when these issues would only surface in postflight off-line

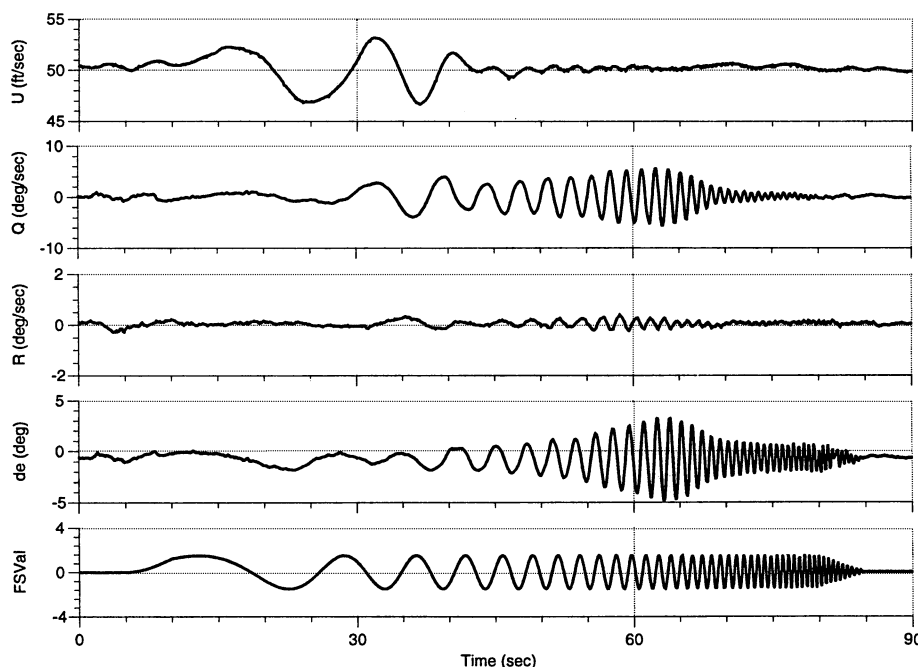


Fig. 5 Typical automated frequency sweep maneuver and vehicle response time histories for Pathfinder where de is longitudinal input and FSVaI is computer-generated sweep.<sup>4</sup>

**Table 1** Hover transfer functions and stability and control derivatives for Fire Scout in hover<sup>7</sup>

Frequency response	Transfer function
$p/\delta_{lat}$ , rad/s · in.	$17e^{-0.043s}/(s + 3.1)$
$q/\delta_{lon}$ , rad/s · in.	$-4.3e^{-0.043s}/(s + 1.0)$
$r/\delta_{ped}$ , rad/s · in.	$-7.7/(s + 0.39)$
$w/\delta_{col}$ , ft/s · in.	$-54/(s + 0.27)$

data processing. Immediate feedback can also be provided to the operator regarding the frequency content of the recorded data and adjustments in the flight-test technique can be made to improve the quality of the flight data.

### Transfer Function Models

Transfer function model identification involves fitting a lower-order equivalent system to a single-frequency response over a specified frequency range to give the closest possible match. This requires first generating the nonparametric frequency responses, specifying the order and form of the transfer function model, selecting the frequency range for the identification, and finally fitting the transfer function to the frequency response. The general transfer function model form is

$$\frac{y}{\delta} = \frac{(b_0 s^m + b_1 s^{m-1} + \dots + b_m) e^{-\tau s}}{s^n + a_1 s^{n-1} + \dots + a_n} \quad (2)$$

where  $\delta$  is the input,  $y$  is the output, and  $\tau$  is the time delay.

It is often the case that the order and form of the transfer functions has to be adjusted to capture the important modes that influence the response in the frequency range selected. This identification procedure is repeated for each frequency response and is typically applied only to the on-axis responses.

As part of the Fire Scout development program, on-axis transfer functions of the angular rate and vertical velocity responses were identified in hover and forward flight conditions from flight data.<sup>7</sup> For this vehicle, the frequency range of interest for flight control work was determined to be 1–10 rad/s. In this frequency range, the vehicle response is dominated by the dynamics of the rigid body, and it was not necessary to include explicit degrees of freedom for the rotor or inflow. Instead, an equivalent time delay  $\tau$  is added to the pitch and roll transfer functions to account for the effect of the higher frequency articulated rotor dynamics on the frequency responses up to 10 rad/s.

The identified hover transfer functions are shown in Table 1. A time delay value of  $\tau = 43$  ms is identified for both the pitch and roll channels independently, which results directly from the lag in the response of the rotor. This time delay of  $\tau = 43$  ms is typical of rotor time constant values for helicopters with this type of rotor system, which adds to the reliability of the identification with a physically meaningful parameter. The bare-airframe transfer functions for Fire Scout in hover and forward flight (shown in Ref. 7) were used in the initial development and optimization of the autonomous control law gains.

### State-Space Models

Whereas simple single-axis transfer functions can accurately characterize the on-axis angular rate and vertical velocity responses, multi-input/multi-output (MIMO) state-space models are needed to characterize fully the coupled air vehicle dynamics. The general form of the state-space model as implemented in CIFER is

$$\dot{\mathbf{M}}\mathbf{x} = \mathbf{F}\mathbf{x} + \mathbf{G}\mathbf{u}, \quad \mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{J}\mathbf{u} \quad (3)$$

The basic equations of motion for a linear model of air vehicle dynamics are based on the Newton–Euler equations for a rigid body that is free to rotate and translate simultaneously in all six degrees of freedom. Depending on the application of the dynamic model, additional dynamics may have to be represented in the linear model to characterize fully the important dynamics of the vehicle. For instance, in modeling helicopters, it is often necessary to include equations for the dynamics of the rotor and inflow, especially for high bandwidth control applications.

**Table 2** Hover transfer functions and stability and control derivatives for Fire Scout in hover<sup>7</sup>

Frequency response	Transfer function	Example state-space model parameters
$p/\delta_{lat}$ , rad/s · in.	$17e^{-0.043s}/(s + 3.1)$	$L_{\delta lat} = 18.1$ $L_p = -3.16$
$q/\delta_{lon}$ , rad/s · in.	$-4.3e^{-0.043s}/(s + 1.0)$	$M_{\delta lon} = -4.46$ $M_q = -0.90$
$r/\delta_{ped}$ , rad/s · in.	$-7.7/(s + 0.39)$	$N_{\delta ped} = -8.93$ $N_r = -0.34$
$w/\delta_{col}$ , ft/s · in.	$-54/(s + 0.27)$	$Z_{\delta col} = -54.0$ $Z_w = -0.26$

State-space model identification is the most complicated part of system identification and requires significantly higher investment in time to obtain dynamics models. In contrast to transfer-function identification where parametric models are fitted to single-frequency responses, state-space model identification involves simultaneously fitting a single model to a number of frequency responses. The steps involved in state-space model identification include 1) generation of the MIMO frequency responses from the flight data, 2) selection of frequency response set to be used in the identification, 3) determination of the model structure to include all of the important dynamics of the vehicle in the desired frequency range, 4) iteration process to fit the model to the frequency responses, and 5) time-domain verification of the final model.

For the Fire Scout program,<sup>7</sup> six-degree-of-freedom (DOF) state-space models were identified in hover and forward flight. These six-DOF models include a time delay on each input channel to account for the rotor and higher frequency dynamics that are not explicitly accounted for in the state-space models. The resulting model was able to capture the off-axis coupling between the roll response to longitudinal and pedal inputs, the pitch response to pedal inputs, and the yaw response to collective inputs. This is in contrast to the transfer function models, which include only the on-axis responses. This state-space model identification was completed two months after the completion of the flight tests.

Table 2 compares the on-axis transfer function models with the on-axis stability derivatives, which constitutes a subset of the total state-space model. The results here show that there is good agreement between the on-axis state space model derivatives and the transfer function control and damping derivatives for each case. This shows that accurate on-axis dynamics characteristics can be generated very quickly using transfer function identification, but to characterize fully the coupled air vehicle dynamics, a state-space model is required. Figure 6 shows the comparison between the state-space model prediction and flight data for a longitudinal doublet as part of the time-domain verification of the dynamic model. The state-space model shows an accurate prediction of both the on- and off-axis responses. This level of fidelity in prediction could not be achieved using the transfer function identification results.

For other RUAV applications, the rotor and inflow dynamics often play an important role in the dynamics of the vehicle in the frequency band that is important for flight control work. Two examples where these additional dynamics had to be modeled explicitly are the BURRO and R-50 vehicles.

The Kaman K-MAX (used for BURRO)<sup>5,6</sup> features an intermeshing twin main rotor system that introduces additional couplings that are not present in conventional helicopters and results in increased dynamic inflow of the rotor system. These rotor and inflow effects have to be accounted for in the dynamic model of the K-MAX. The choice of model structure is based on the key dynamics in the frequency range of interest for flight control work on this vehicle (0.3–12.0 rad/s). For the cases where the vehicle was unloaded, the model was composed of the rigid-body states ( $u$ ,  $v$ ,  $w$ ,  $p$ ,  $q$ ,  $r$ ,  $\phi$ , and  $\theta$ ) and coupled coning-inflow dynamics, resulting in a total of eight-DOF. In addition, a time delay is included to account for the unmodeled higher frequency dynamics such as rotor, linkage, and instrumentation effects. Figure 7 shows an example frequency-response fit of the identified model for the unloaded vehicle in hover.

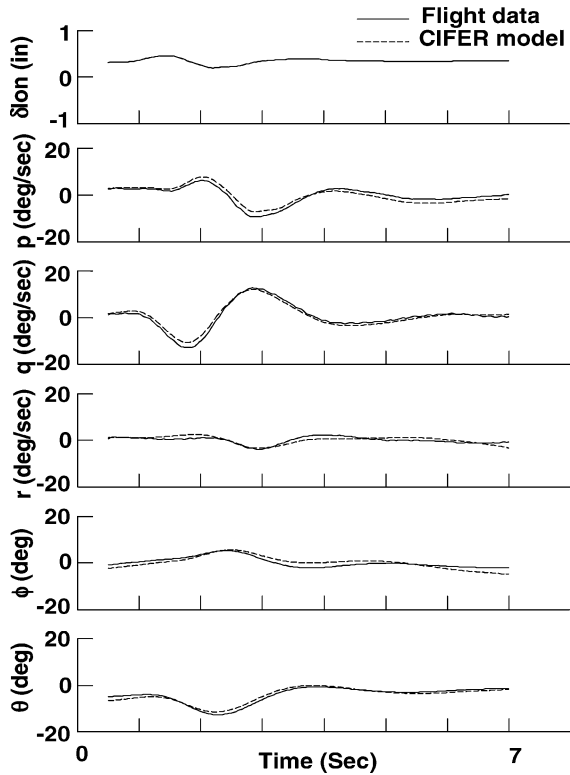


Fig. 6 Time-domain verification of Fire Scout hover state-space model with longitudinal doublet.<sup>7</sup>

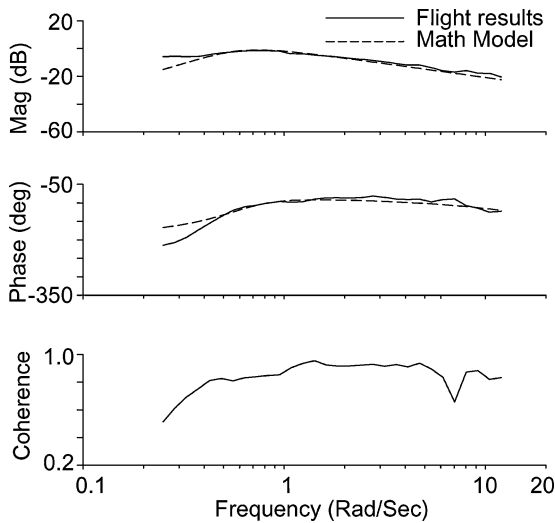


Fig. 7 Roll rate to lateral control frequency response for BURRO in hover (unloaded case).<sup>5</sup>

This is the bare-airframe roll rate frequency response to lateral control. Good coherence is achieved in the required frequency range indicating good quality flight data.

For the loaded case, the external load was explicitly accounted for in the dynamic model with the addition of analytical equations for the load. Additional DOF for the lateral trolley position and the longitudinal cable angle were also included. Figure 8 shows the geometry for the two-body problem of the BURRO with an external load. (Additional details of the geometry may be found in Ref. 6.) For the lateral motion of the cable, the hook assembly is suspended from a small trolley, which is free to roll along a curved track under the belly of the aircraft. This mechanism decouples the load motion from the body motion because the lateral force from the cable passed through the center of gravity of the body. The addition of the load equations and cable DOF adds additional complexity to the model

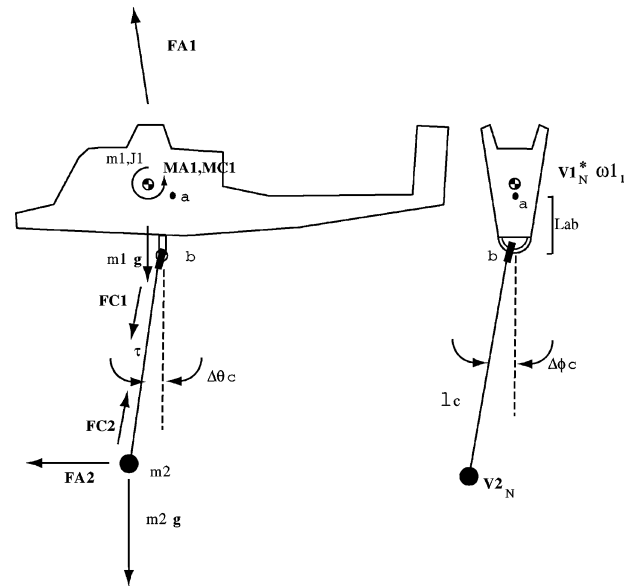


Fig. 8 Geometry and DOF for BURRO with an external slung load attached.<sup>6</sup>

and to the identification. The form of the state-space model is as follows:

$$\begin{bmatrix} I_{5 \times 5} & 0_{5 \times 8} & 0_{5 \times 4} \\ 0_{8 \times 5} & I_{8 \times 8} & -\partial \dot{u} / \partial \dot{x}_r \\ 0_{4 \times 5} & 0_{4 \times 8} & M_r \end{bmatrix} \begin{pmatrix} \delta \dot{q} \\ \delta \dot{u} \\ \delta \dot{x}_r \end{pmatrix} = \begin{bmatrix} 0_{5 \times 5} & \partial \dot{q} / \partial u & 0_{5 \times 4} \\ \partial \dot{u} / \partial q & \partial \dot{u} / \partial u & 0_{8 \times 4} \\ 0_{4 \times 5} & 0_{4 \times 8} & F_r \end{bmatrix} \begin{pmatrix} \delta q \\ \delta u \\ \delta x_r \end{pmatrix} + \begin{bmatrix} 0_{5 \times 4} \\ \partial \dot{u} / \partial \delta \\ G_r \end{bmatrix} \begin{pmatrix} \delta_{lat} \\ \delta_{lon} \\ \delta_{ped} \\ \delta_{col} \end{pmatrix} \quad (4)$$

(Additional details of the state-space model formulation may be found in Ref. 6.) The first five equations represent the dynamics of the external load, the next eight are the rigid-body equations of motion of the vehicle, and the final four equations are for the coupled inflow-coning dynamics. Note that the results for the BURRO vehicle with a slung load represent the first state-space model identification of a coupled helicopter-slung load configuration. (Identification results with a slung load may be found in Ref. 6.)

As with the K-MAX, the dynamics of the rotor system for the R-50 are important to the low-frequency response of the vehicle and must be explicitly accounted for in the dynamic model.<sup>8,9</sup> The R-50 features a two-bladed teetering rotor with a Bell-Hiller stabilizer bar. The dynamic response of the vehicle is dominated by the rotor system and the stabilizer bar, which have to be explicitly accounted for in the dynamic model of the vehicle. Another key aspect of the dynamics of the R-50 is the coupling between the main rotor and the helicopter fuselage. Figure 9 shows the on-axis roll rate response to lateral control, and this response exhibits a mode at 11 rad/s that is due to the coupling between the rotor system and fuselage. The degree of this coupling is indicated by the value of the equivalent flap spring stiffness normalized by the inertia of the vehicle,<sup>13</sup> with higher values producing more coupling. For the R-50 with a teetering rotor system, the normalized equivalent flap spring stiffness is higher than that typical of full-scale helicopters, and the R-50, therefore, exhibits more rotor-fuselage coupling. For medium-to-high bandwidth control synthesis, this coupling must be included in the helicopter model because most of the control forces and moments are produced by the rotor response. In addition to equations for the stabilizer bar, the R-50 model includes explicit equations for the rotor dynamics, which are coupled to the fuselage equations of motion. Figure 9 shows that the identified model response that includes explicit equations for the rotor dynamics tracks very well with the response derived from flight data.

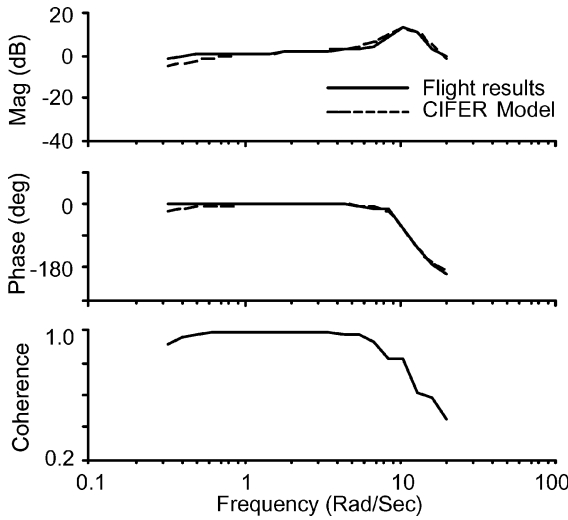


Fig. 9 Roll rate to lateral stick frequency response for R-50 in hover.<sup>9</sup>

The final model structure for the R-50 consists of 13 states. (Additional details of the state-space model for the R-50 may be found in Ref. 9.) These states include eight rigid-body states, four coupled rotor-fuselage states (two for the main rotor and two for the stabilizer bar), and one state for the yaw stability augmentation system, which was turned on during the flight tests. Equation (5) shows the form of the parameterized state-space model for hover. The reason for the complicated model is that the response is dominated by the rotor because the body has low mass and inertia and any model that is used for flight control development and optimization for the R-50 must include the coupled dynamics of the rotor and fuselage.

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{p} \\ \dot{q} \\ \dot{\phi} \\ \dot{\theta} \\ \tau_f \dot{a} \\ \tau_f \dot{b} \\ \dot{w} \\ \dot{r} \\ \dot{r}_{fb} \\ \tau_s \dot{c} \\ \tau_s \dot{d} \end{bmatrix} = \begin{bmatrix} X_u & 0 & 0 & 0 & 0 & -g & X_a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_v & 0 & 0 & 0 & g & 0 & 0 & Y_b & 0 & 0 & 0 & 0 \\ L_u & L_v & 0 & 0 & 0 & 0 & 0 & L_b & L_w & 0 & 0 & 0 & 0 \\ M_u & M_v & 0 & 0 & 0 & 0 & M_a & 0 & M_w & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\tau_f & 0 & 0 & -1 & A_b & 0 & 0 & 0 & A_c & 0 \\ 0 & 0 & -\tau_f & 0 & 0 & 0 & B_a & -1 & 0 & 0 & 0 & 0 & B_d \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_a & Z_b & Z_w & Z_r & 0 & 0 & 0 \\ 0 & N_v & N_p & 0 & 0 & 0 & 0 & 0 & N_w & N_r & N_{r_{fb}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_r & K_{r_{fb}} & 0 \\ 0 & 0 & 0 & -\tau_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -\tau_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} u \\ v \\ p \\ q \\ \phi \\ \theta \\ a \\ b \\ w \\ r \\ r_{fb} \\ c \\ d \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & Y_{ped} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{col} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ A_{lat} & A_{lon} & 0 & 0 \\ B_{lat} & B_{lon} & 0 & 0 \\ 0 & 0 & 0 & Z_{col} \\ 0 & 0 & N_{ped} & N_{col} \\ 0 & 0 & 0 & 0 \\ 0 & C_{lon} & 0 & 0 \\ D_{lat} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{lat} \\ \delta_{lon} \\ \delta_{ped} \\ \delta_{col} \end{bmatrix} \quad (5)$$

### Subsystem Model Identification

In addition to the identification of the bare airframe vehicle dynamics, the subsystem dynamics must also be characterized because they contribute significantly to the overall dynamic performance and stability of the vehicle. Subsystems encompass all of the components in the overall control system block diagram (Fig. 2) apart from the bare airframe of the vehicle. The digital flight control computer typically adds a time delay  $\tau$  to the feedback system that is dependent on the update rate of the computer. Sensors add dynamics and time delays to the system, particularly from filtering and digitizing the sensor measurements. Actuators are typically characterized by a bandwidth and damping as well as nonlinear rate and position limits.

For UAVs, subsystem dynamics are particularly important because there are usually many trade-offs involved in the selection of

the subsystem components. In general, unmanned vehicle designs and components are not held to the same strict guidelines and regulations as manned vehicles, and lower performance components are often used because of their availability, lower cost, reduced weight and size, etc. In particular, for small-scale UAVs, the dynamic responses of the vehicle become faster as the vehicle designs get smaller. At the same time, the performance of the flight control sensors, flight control computer, and actuators degrades because of restrictions in available volume and weight and the need to reduce system cost. In many cases, the performance of the subsystem components proves to be the limiting factor in the overall system stability and performance.

It is possible to rely on the manufacturers' specifications for the dynamics and delays associated with the subsystem components. This is not always a wise choice because manufacturers' specifications are often taken in ideal test conditions that do not translate well an operational environment, are often an average where individual articles may not meet the specifications, and may well be exaggerated for marketing purposes. A better alternative is to use a frequency-domain identification technique to identify the dynamics and delays of actuators, sensors, and the flight computer.

An example of this technique is the development of the flight control system for the BURRO vehicle.<sup>8,9</sup> For the initial flight control work, the sensor dynamics were modeled as equivalent time delays based on the manufacturers' specifications for each sensor component. The actuators were bench tested outside the vehicle, and frequency-domain system identification was used to extract transfer functions of the actuator responses. When the vehicle dynamics model obtained from system identification and the sub-system models was used, the control system gains were tuned to satisfy a set of stability, handling qualities, and performance specifications. However, flight tests with these optimized control law gains showed vehicle responses that were significantly different from those calculated

from the model. This is shown in Fig. 10, where a roll oscillation is measured in flight that was not predicted with the identified model and optimized control gains.

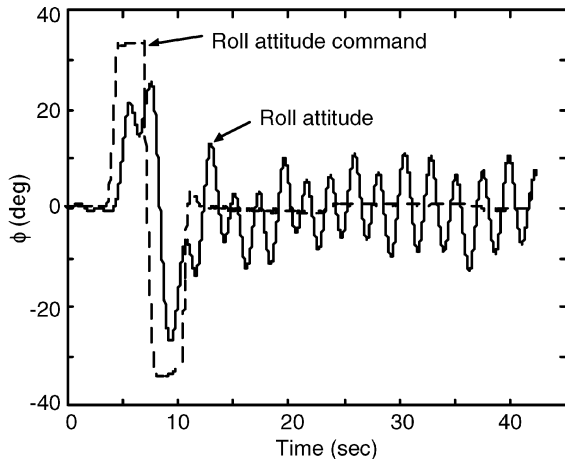
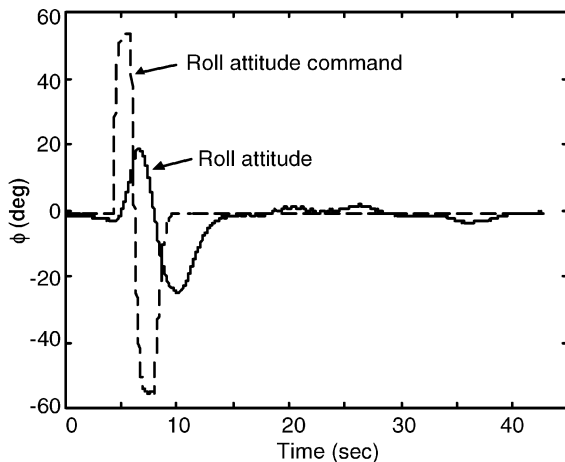
To identify the source of this difference, a series of doublets in each axis were flown with the closed-loop vehicle, and frequency responses were extracted at various points in the control system. The availability of these frequency responses allowed for the accurate identification of the dynamics and time delays associated with the vehicle subsystems. It was found that the total time delay in the system was significantly higher than first thought, which contributed to the differences in the responses. Table 3 shows the identified time delays and compares them with the specifications.

These new time delays were used to reoptimize the control law gains, which produced a much improved correlation with the flight-test measurements. The flight data for a lateral doublet with the



**Table 3 Comparison of estimated and actual delays for BURRO<sup>8</sup>**

Component	Estimated delay, ms	Actual delay, ms
Actuators	50	107
Sensors	25	53
Computer	20	40
Filters	0	90
Total	95	290

**Fig. 10 Oscillatory roll response with initial gains for BURRO in hover.<sup>6</sup>****Fig. 11 Improved flight-test roll response with updated control law gains for BURRO in hover.<sup>6</sup>**

updated control law gains is shown in Fig. 11, where the unstable roll oscillation is no longer present.

With the BURRO UAV, the accurate characterization of the subsystem dynamics was critical to the design and optimization of the flight control system. Frequency-domain system identification was successfully used to characterize accurately the dynamics and time delays of the various subsystems in the control loop, which significantly improved stability and performance of the vehicle.

### Conclusions

1) The analysis and design of flight control systems for UAVs pose some unique challenges that are not seen when dealing with manned vehicles. Highly accurate models of the vehicle responses are required because UAVs are often tasked to fly autonomously in difficult weather conditions and close to obstacles and populated areas. Difficulties arise in using analytical methods based on first principles due to the short development times and reduced cost of many UAV programs. In addition, many small-scale and unconventional designs are used, which add to modeling and control difficulties.

2) The frequency-domain system identification method is well suited to the modeling of UAVs as an alternative to or complementary with analytical modeling methods. With many UAV programs, flight hardware is available early in the program, and with the flight vehicle available, dynamic response models of the vehicle can be extracted and validated rapidly from flight data. The system identification method also allows for rapid updating of vehicle response models as physical changes are made to the vehicle configuration. System identification results can also be used to validate and improve analytical simulation models.

3) System identification techniques have also been applied to the vehicle subsystems including the sensors and actuators. These subsystems must be modeled correctly because they contribute significantly to the overall dynamic performance and stability of the vehicle. System identification provides an alternative to relying on manufacturers specifications for subsystem models, which is important because manufacturers' specifications are not always accurate. This is particularly important for UAVs because these vehicles are more likely to use components where the specifications are not known or less precisely known.

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